

Application of the Separate Universe Approach to Preheating

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Abstract

The dynamics of preheating after inflation has not been clearly understood yet. In particular, the issue of the generation of metric perturbations during preheating on super-horizon scale is still unsettled. Large scale perturbations may leave an imprint on the cosmic microwave background, or may become seeds for generation of primordial black holes. Hence, in order to make a connection between the particle physics models and cosmological observations, understanding the evolution of super-Hubble scale perturbations during preheating is important. Here, we propose an alternative treatment to handle this issue based on the so-called separate universe approach, which suggests less efficient amplification of super-Hubble modes during preheating than was expected before. We also point out an important issue which may have been overlooked in previous treatments.

1 Introduction

Rather recently, people realized that the process of reheating after inflation need not simply be described by the usual single particle decay of the inflaton field. If there are relevant scalar field degrees of freedom coupled to the inflaton field at the end of inflation, the coherent oscillation of the inflaton can excite those scalar fields by parametric resonance[1]. This process was extensively investigated, in most cases, neglecting metric perturbations, which typically will be a good approximation as long as we are interested in processes occurring within the Hubble horizon scale. Often the energy transfer due to parametric resonance is much more efficient than that due to the usual particle decay, and sometimes it is violent. It was then suspected that there might be significant effects on the large scale perturbations beyond the Hubble scale[2] due to resonance. Some particle physics models may predict distinguishable signals in the cosmic microwave background, or formation of inconsistently large number of primordial black holes [3].

Here in this short contribution, we would like to propose another approach to discuss this issue of the generation of large scale perturbations at preheating. We make use of the separate universe approach [4]. We stress that unphysical acausal energy transfer, which may not carefully prohibited in previous treatments, does not take place in this new approach.

2 Separate universe approach

Basic Idea : We consider the system composed of many scalar fields. The discussions in this paper is based on the following statement obtained in Ref. [5, 4, 6].

When we solve the background spatially homogeneous field equation, the e -folding number $N \equiv \log a$ until the expansion rate H reaches a given value H_* depends on the initial value of the fields Φ (and their time derivatives). Let's denote this value of N by N_{H_*} . Suppose that the initial value of Φ has spatial variation over super-Hubble scales. Then, the value of N_{H_*} becomes a function of the spatial position \mathbf{x} . In this case, the newly generated curvature perturbations on the constant Hubble slice \mathcal{R} is simply given by ΔN_{H_*} , i.e., the fluctuations in N_{H_*} .

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Mechanism of Generating Curvature Perturbations from Isocurvature Perturbations :

Let's consider matter fields consisting of two components. We call them ϕ and χ . They are not necessarily scalar fields here. We assume that the rates of change of the energy density of ϕ and χ due to the expansion of the universe are different. Namely,

$$\frac{d \log \rho_\phi}{dN} = -3(1 + w_\phi), \quad \frac{d \log \rho_\chi}{dN} = -3(1 + w_\chi), \quad (1)$$

For example, the energy density of ϕ changes like dust matter, while χ behaves as a radiation fluid. In this case, we have $w_\phi = 0$ and $w_\chi = 1/3$. Let us denote the fraction of the energy density of χ by $f := \rho_\chi / \rho$. Then, we have

$$\frac{d \log \rho}{dN} = -3(1 + w_\phi) + 3(w_\phi - w_\chi)f. \quad (2)$$

From this expression, we notice that there are two factors necessary for generation of super-Hubble adiabatic perturbations. One is non-vanishing $w_\phi - w_\chi$, and the other is inhomogeneity of f .

3 Conformal two field model

Model : We consider the model whose potential is given by

$$V = \frac{\lambda}{4}\phi^4 + \frac{1}{2}g^2\phi^2\chi^2. \quad (3)$$

When $g^2 = 2\lambda$, the longest wavelength mode of χ is the fastest growing mode, and the initial spectrum of χ is scale-invariant. This appears to lead to exponential growth of pre-existing entropy perturbations even when backreaction is taken into account [7]. The equations of motion for the spatially homogeneous background are given by

$$\ddot{\Phi}^i + 3H\dot{\Phi}^i - \frac{dV}{d\Phi^i} = 0, \quad H^2 = \frac{\kappa^2}{3} \left(\frac{1}{2}\dot{\Phi}^2 + V \right). \quad (4)$$

Here we used the vector notation to represent the two component scalar fields, ϕ and χ , simultaneously. Namely, $\Phi^1 = \phi$ and $\Phi^2 = \chi$.

Simplified Analysis with Time Averaging : We define the averaged energy in a unit comoving volume by $\mathcal{E} = \left\langle a^3 \left(\frac{1}{2}\dot{\Phi}^2 + V \right) \right\rangle$, where $\langle \cdots \rangle$ represents time averaging. Let us assume that $\dot{\Phi}^2$ is bounded and is oscillating. Then, we have many zeros of $\dot{\Phi} \cdot \dot{\Phi}$. If we take the average between two zeros, we have

$$0 = \left\langle \frac{d}{dN} \left(a^3 \dot{\Phi} \cdot \dot{\Phi} \right) \right\rangle = \left\langle \frac{a^3}{H} \left(\dot{\Phi}^2 - \lambda\phi^4 - 2g^2\chi^2\phi^2 \right) \right\rangle. \quad (5)$$

Since H is slowly changing, we obtain an approximate relation: $\left\langle a^3 \left(\dot{\Phi}^2 - \lambda\phi^4 - 2g^2\chi^2\phi^2 \right) \right\rangle = 0$. Therefore we have

$$\frac{d \log \mathcal{E}}{dN} = 3 - 3 \frac{\left\langle a^3 \dot{\Phi}^2 \right\rangle}{\mathcal{E}} \approx -1. \quad (6)$$

From this fact, $\mathcal{E} \approx e^{-N}$ is deduced, and hence we have $H^2 \propto \rho \approx \mathcal{E}/a^3 \propto a^{-4}$. Anyway, N_{H_*} is basically independent of the initial value of the field. Therefore it seems that additional curvature perturbations are not generated when the wavelength of a mode is longer than the horizon scale. In the language of the preceding section, $w_\phi - w_\chi$ vanishes approximately in this model. Hence, amplification of fluctuations of $O(1)$ is hardly expected. Notice that H^2 is roughly proportional to Φ^4 but the mass squared is to Φ^2 . Hence, as the amplitude of Φ decays, the mass squared tends to be larger than H^2 .

Detailed Analysis without Time Averaging : We solve the above set of equations numerically varying the initial value of χ . Then, we can evaluate the initial value dependence of N_{H_*} , the e -folding

number until the total energy density reaches a certain specified value. However, the original model has the following shortcoming. As there are only two degrees of freedom corresponding to the homogeneous modes of ϕ and χ -fields, the exchange of energy between them continues for ever. Then, occasionally the value of ϕ becomes exceptionally small. In this phase, the period between two neighboring zeros of $\dot{\Phi} \cdot \dot{\Phi}$ becomes very long. If the specified target energy density is reached during this phase, ΔN becomes largely dependent on the choice of the final value of the Hubble rate H_r . Then, interpretation of the results becomes difficult. In more realistic situations, the energy density carried by these long-wavelength modes will be redistributed to a large number of other degrees of freedom (shorter wavelength modes of these or other fields) sooner or later through rescattering.

Here we assume two radiation fluids, r_ϕ and r_χ . Φ^i decays into the component r_i with decay constant Γ_i . The modified set of equations are given by

$$\ddot{\Phi}^i + (3H + \Gamma_i)\dot{\Phi}^i - \frac{dV}{d\Phi^i} = 0, \quad \dot{\rho}_{r_i} = 4H\rho_{r_i} + \Gamma_i(\Phi^i)^2, \quad H^2 = \frac{\kappa^2}{3} \left(\frac{1}{2}\dot{\Phi}^2 + V + \sum_{i=1}^2 \rho_{r_i} \right), \quad (7)$$

where ρ_{r_i} is the energy density of the i -th component of radiation fluids.

We consider the case with $g^2 = 2\lambda = 0.02$ and $\Gamma_\phi = \Gamma_\chi = 10^{-5}$. We set the initial condition at $\phi = 1$ with $\dot{\phi} = \dot{\chi} = 0$. Using the convention $\kappa^2/3 = 1$, the initial Hubble rate is $H = 0.05$, where we assume a negligible contribution to the initial energy density from the χ -field. We integrate Eqs. (7) until the Hubble rate goes down to 10^{-6} , at which point the energy is almost completely transferred to the radiation fluids. In Fig. 1, we give a plot of the e -folding number N between $H = 0.05$ and $H = 10^{-6}$ varying the initial value of χ . We see small fluctuations of the e -folding number N depending on the initial value of χ . Each plot contains only 100 sampling points with equal horizontal spacing. The amplitude of fluctuation is not $O(1)$ but still it is significant compared to that observed in the cosmic microwave background. An interesting and important point is that the dependence on the initial value of χ of this fluctuation is almost random. Two panels in Fig. 1 are plots with different range of the initial value of χ . Although the scale of the horizontal axis is very different, these two plots look quite similar.

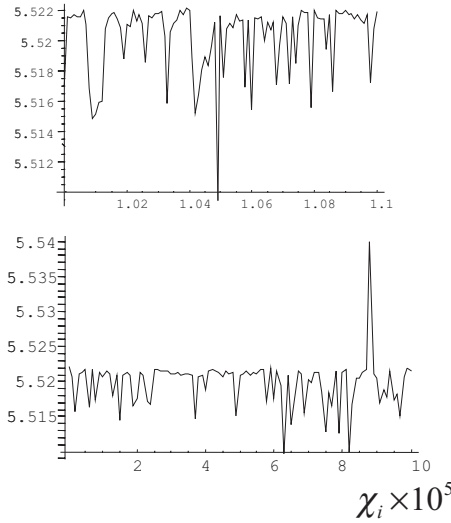


Fig.1: Plots showing the initial value dependence of the e -folding number N . In each panel 100 discrete points are plotted. The scale of the horizontal axis is different, but these two plots look quite similar. This will mean that the distribution of N is rather random.

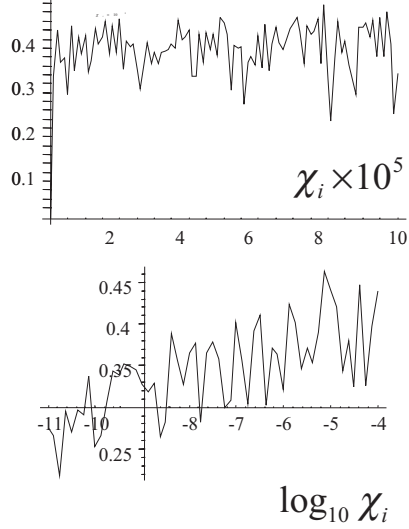


Fig.2: Similar plots to Fig.1 but the vertical axis is the fraction of the radiation energy originating from the secondary field χ . The panel (b) is a close-up view of the panel (a) for small initial value of χ . We can see a trend that the fraction increases as the initial value of χ increases.

The origin of these fluctuations can be understood as follows. If the oscillation of fields is fast compared to the change rate of H , the approximation using the time averaging must be good. The error will be exponentially suppressed. As mentioned above, however, the period of oscillation occasionally becomes very long if the motion of the scalar fields is trapped by chance in the region where the amplitude of oscillation in the direction of ϕ -field is exceptionally small. In such situation, the approximation we

made before is no longer valid. Since the motion of the two scalar fields are rather chaotic, the initially neighboring trajectories in the configuration space of ϕ and χ soon deviate. Thus, it is quite sensitive to the tiny difference in the initial condition whether this entrapment occurs or not. As a result the dependence on the initial condition is almost completely randomized.

Then, the question is whether this amplification of fluctuations can be a process that generates significant fluctuations at the super-Hubble scale during preheating? Since the amplitude of the generated curvature perturbation is so sensitive to χ_i , the variance seen in Fig. 1 will basically represent the variance of random fluctuations incoherent beyond the Hubble scale. If the fluctuations are completely random beyond a certain cutoff scale k_c^{-1} , we have $n = 4$ spectrum for $k < k_c$ in general. (Here we use the notation that $n = 1$ corresponds to the scale invariant spectrum.)

However, it is not certain whether a smoothing of $N(\chi)$, e.g., $\tilde{N}(\chi_i) := \int d\chi N(\chi) W(\chi - \chi_i)$ with an appropriate window function W is tilted or not. Hence, the averaged value on very large scale may have an additional fluctuation other than the $n = 4$ random component. The amplitude of this additional component is difficult to determine because we need to reduce the contamination of $n = 4$ component. To do so, we need to compute the average over sufficiently large volume. This means that we need to calculate the value of N for sufficiently large number of samples changing the initial value of χ , which we have not done yet. Hence, we have not completely rejected the possibility of generating fluctuation coherent beyond the Hubble scale. However, the amplitude of this coherent fluctuation should be much more suppressed than the incoherent one.

Generation of Curvature Perturbations :

If the exact conformal invariance is broken by some reason, we can expect a curvature perturbation may be generated. The easiest way will be changing the decay process of the two fields. In the above calculation, we have introduced the decay terms by hand assuming that the both scalar fields decay into a radiation fluid with the same decay constant Γ . We distinguished two components of the radiation fluids depending on which scalar field becomes the origin. Suppose, one radiation fluid is composed of massive fields, although the mass is negligible during the preheating epoch. Then, sooner or later, the mass of this radiation fluid becomes larger than the Hubble rate, and the energy density of this field starts to behave like $\propto e^{-3N}$, while the other continues to evolve as $\propto e^{-4N}$. Therefore the isocurvature perturbation imprinted as the ratio $\xi := \rho_{r_\chi} / (\rho_{r_\phi} + \rho_{r_\chi})$, is transformed into the curvature perturbation at this stage.

Now let us look at the plot of this ratio as a function of the initial value of χ . From Fig. (2a), it seems that the dependence on χ_i is just random fluctuations. However, if we close up the portion for small $|\chi_i|$ (Fig. (2b)), we can see clear dependence on χ_i , which can be explained as follows. For small χ_i , the decay to the radiation fluid becomes effective before the equi-partition between the homogeneous parts of the ϕ and χ fields is achieved by the parametric resonance. In this case, the decay to the radiation fluid is more efficiently through the ϕ field. As a result, the ratio ξ becomes small for small $|\chi_i|$. This mechanism to generate large scale fluctuations is very efficient because the fluctuation continues to be amplified until the matter radiation equality. Hence, this process will be a possible mechanism to create primordial black holes.

4 Discussion

In this paper, applying the separate universe approach, we discussed generation of metric perturbations at super-Hubble scale during preheating. As an example, we considered the conformal two scalar model whose potential is given by Eq. (3). Previously, it was argued that large amplitude of fluctuations can be generated during the reheating process, but we found that the separate universe approach suggests relatively suppressed amplitude of fluctuations.

We suspect that the origin of this discrepancy is in an overlooked point in previous analyses. Usually people decompose fields as (homogeneous background)+(perturbations), using the common homogeneous background for separated universes. Let's consider two largely-separated homogeneous patches, universe A and universe B . Suppose that the preheating is efficient in the universe A but inefficient in the universe B . Then, we physically expect that the back reaction to the inflaton oscillation is also efficient in the universe A . However, as long as we use the common homogeneous background, the back reaction works in the same way both in the universe A and in the universe B . This means that artificial and acausal

energy transfer from the universe B to A easily takes place in the treatment using the homogeneous background.

However, the above criticism to the previous works does not apply to the numerical studies. For example, in Ref.[8], it was argued that super-Hubble modes are enhanced even in a single field model. They considered $\lambda\phi^4$ model. When they simulate the case that only a single mode in the resonance band is excited, there was no enhancement in super-Hubble modes. On the other hand, when all the modes in the same lowest resonance band are excited, there was enhancement in super-Hubble modes. Their explanation is as follows. The gauge invariant perturbations at super-Hubble scale are generated by the quadratic product of smaller scale perturbations in the resonance band. Namely, $\zeta_{\mathbf{k}} \propto \int d^3k \chi(\mathbf{k}') \chi(\mathbf{k}-\mathbf{k}')$. This integral becomes approximately constant for small k less than the width of the resonance band Δk . For larger k , this integral is small because, when one of the arguments of χ is in the resonance band, the other is outside of it. So this enhancement mechanism does not apply for small scale modes. In this sense, the large scale modes are selectively enhanced.

However, since $\zeta_{\mathbf{k}} \propto k^{-3/2}$ for the scale invariant spectrum, constant value of $\zeta_{\mathbf{k}}$ does not mean enhancement of larger wavelength mode. Therefore the mechanism of the enhancement of super-Hubble modes works only when the cutoff scale $a\Delta k^{-1}$ is much larger than the Hubble radius. We can easily see that this condition is not satisfied in this model. For the redefined field $\hat{\phi} = a\phi$, the field equation becomes $\hat{\phi}'' + \lambda\hat{\phi}^3 - (a''/a)\hat{\phi} = 0$. Here we used the conformal time coordinate $\eta = \int a^{-1} dt$, and a prime denotes differentiation with respect to η . Since the radiation dominant universe is a good approximation for this model, we use $a = (t/t_0)^{1/2}$. Then, $\eta = 2t_0a = 1/\mathcal{H} = 1/aH$, and therefore $H = 2t_0/\eta^2$. The amplitude of oscillation of $\hat{\phi}$ is constant. We can set the amplitude of $\lambda\hat{\phi}^2$ to unity without loss of generality. In fact, using the Friedmann equation, we have $\lambda\hat{\phi}^2 \approx (8\pi G/3a^2)\lambda\phi_i^4 = \mathcal{H}^2 = 1/(2t_0a)^2$, where we have used the fact that the initial value of ϕ, ϕ_i , is $O(m_{pl})$. Choosing t_0 appropriately, we can set $\eta_i = 1$. This also means that the comoving wave number corresponding to the initial Hubble horizon \mathcal{H} is 1.

The resonance effect becomes important after $\mu\eta$ becomes $O(1)$, where μ is the growth rate of the amplitude due to parametric resonance. Hence, $\eta \approx \mu^{-1}$ when the parametric resonance becomes important. At this epoch, the comoving horizon scale is $\mathcal{H} = \eta^{-1} \approx \mu$. Then, the condition for the enhancement of the super-Hubble modes will be $\mu \gg \Delta k$. In this model $\mu < 0.267$ and $\Delta k \approx 0.2$. Therefore the condition is not satisfied.

However, the plots in Ref.[8] show steep increase of power for small k . The expected flat spectrum, which does not mean scale invariance, is not seen. This is no contradiction. When they plot the figures, they plot only discrete numbers of k modes and the plots do not have resolution less than Δk . This can be seen by looking at the shape of the resonance peak, which does not show the expected round shape. This means that the resolution is less than Δk . Since the mode $k \approx \Delta k$ is inside the Hubble scale after the resonance becomes efficient, actually only one mode outside the horizon scale is plotted in those figures. Therefore it is not strange that the expected plateau is absent in their plots. As mentioned before, this $n = 4$ spectrum is common in the situation that there are random fluctuations of a large amplitude on small scales and seems to refute the claims of [7].

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